

Renormalization in large- N QCD is incompatible with open/closed string duality

Marco Bochicchio^a

^a*INFN sez. Roma 1*

Piazzale A. Moro 2, Roma, I-00185, Italy

E-mail: marco.bochicchio@roma1.infn.it

ABSTRACT: Recently, we demonstrated in [Phys. Rev. D **95**, 054010](#) that the renormalization group and asymptotic freedom (AF) imply that the Yang-Mills (YM) S matrix is ultraviolet (UV) finite in the 't Hooft large- N expansion, while the S matrix in large- N QCD with massless or massive quarks, and in QCD-like theories, is only renormalizable because of log divergences of meson loops starting from the order of $\frac{N_f}{N}$ arising from the change of β_0 at that order. We investigate the compatibility of such seemingly innocuous renormalization properties with the existence of a would-be canonical string solution, admitting open/closed duality, matching the 't Hooft large- N expansion in YM, QCD and QCD-like theories. The UV finiteness of the YM S matrix is compatible with the universally believed UV finiteness of closed-string diagrams, but open/closed duality turns out to be incompatible with the UV divergence of glueball amplitudes with the insertion of one meson loop in large- N QCD. Naively, the incompatibility arises because such UV-divergent one-loop amplitudes are dual to tree closed-string diagrams in the pure YM theory, which are universally believed to be both UV finite – since they are closed-string tree diagrams – and infrared finite because of the glueball mass gap. In fact, we resolve this issue by means of a low-energy theorem of the Novikov-Shifman-Vainshtein-Zakharov type derived in [Phys. Rev. D **95**, 054010](#) that controls the renormalization in QCD-like theories both perturbatively and nonperturbatively in the large- N expansion. It turns out that the low-energy theorem is compatible, in a perturbatively massless QCD-like theory, with open/closed duality, perturbatively at order of g_{YM}^2 – because of conformal symmetry and absence of mass gap – but it is incompatible with the duality, in QCD with massless or massive quarks, nonperturbatively at order of $\frac{N_f}{N}$ because of the AF and the aforementioned change of β_0 . The incompatibility extends to the large- N 't Hooft expansion of a vast class of confining asymptotically free QCD-like theories including $\mathcal{N} = 1$ SUSY QCD.

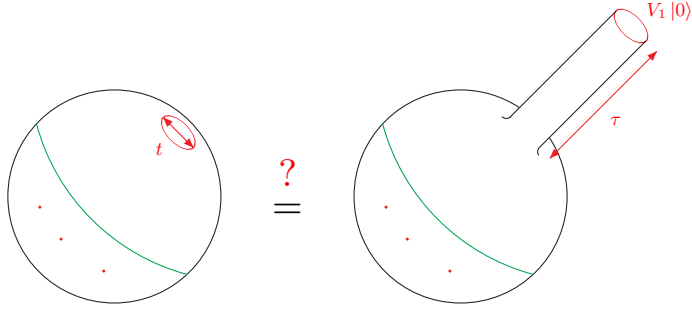


Figure 1. If a canonical string solution of large- N QCD exists, the one-loop open-string diagram on the lhs is a cutoff-independent volume form on the moduli that, by integrating on the small- t region, must be UV log divergent because of the renormalization properties of large- N QCD. By open/closed duality, the lhs coincides, as a volume form on the moduli, with the tree closed-string diagram in the planar pure YM theory on the rhs, where the cylinder has the length $\tau = f(t) \sim \frac{1}{t}$. Thus, the rhs must be divergent, but only by integrating on τ , that is incompatible with the universal belief that is both UV finite – since it is a tree closed-string diagram – and IR finite because of the glueball mass gap. In fact, the low-energy theorem in large- N QCD implies that the rhs is log divergent before integrating on τ because of the explicit UV log divergence of the operator counterterm, V_1 , in the boundary state, $V_1 |0\rangle$. Hence, the rhs cannot coincide as a volume form on the moduli with the lhs, and open/closed duality cannot hold in large- N QCD.

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1 Introduction and conclusions

Recently, we studied the renormalization properties [1] of the large- N 't Hooft [2] and Veneziano [3] expansions both in Yang-Mills (YM) theory, QCD with massless or

massive quarks, and QCD-like theories, implied by the renormalization group (RG) and asymptotic freedom (AF).

A main result in [1] is that RG and AF imply that the 't Hooft large- N expansion of the YM S matrix is ultraviolet (UV) finite, while the 't Hooft expansion of the QCD S matrix is only renormalizable due to UV log divergences of meson loops starting from the order of $\frac{N_f}{N}$: they arise from the log-divergent renormalization of Λ_{QCD}^P , the 't Hooft planar RG invariant [1], because of the additive correction [1], β_0^{NP} , at order of $\frac{N_f}{N}$ to the first coefficient of the beta function, $\beta_0 = \beta_0^P + \beta_0^{NP}$.

The aim of this paper is to investigate the implications of these seemingly innocuous renormalization properties for the existence of a would-be canonical string solution [3–5], satisfying open/closed string duality [6] (Sec. 2), of the large- N 't Hooft expansion in both YM theory, QCD and QCD-like theories.

Surprisingly, it turns out that the open/closed duality of the would-be string solution is incompatible with the renormalization and glueball mass gap in large- N QCD and QCD-like theories.

As a consequence, a deep reconsideration of the string program for QCD and QCD-like theories, that has been lasting for forty years (Sec. 2), is required. Heuristically, in [7, 8] we outlined a proposal for a noncanonical way-out. We leave for the future a similar investigation about the Veneziano expansion.

We explain now why the issue about the incompatibility inevitably arises as an immediate consequence of the aforementioned renormalization properties.

The renormalization properties on the gauge side can be translated in terms of a supposed canonical string solution (Sec. 2) matching the 't Hooft topological expansion of large- N QCD, involving open-string diagrams, in which mesons propagate, and closed-string diagrams, in which glueballs propagate.

The UV finiteness of the large- N YM S matrix [1] matches the universal belief that all consistent closed-string theories are UV finite [9, 10]. This matching is remarkable, since two very different and largely independent features imply the finiteness: on the gauge side, AF and RG [1], on the string side, conformal symmetry on the string world-sheet and, as it is universally believed [9, 10], modular invariance of closed-string amplitudes.

However, it is unclear whether this is a deep fact or just a coincidence since the very same conformal symmetry implies open/closed string duality [6] that turns out to be incompatible with the aforementioned renormalizable UV divergences in the large- N QCD S matrix as we will show in Sec. 4.

For example, the following issue arises: an implication of open/closed duality [6] is the equality, within the leading $\frac{1}{N}$ accuracy, between (amputated on-shell) glueball k -point correlators (Sec. 2):

$$\langle \text{Tr } F^2 \dots \text{Tr } F^2 \rangle_{conn}^{1-OpenStringLoop} = \langle \text{Tr } F^2 \dots \text{Tr } F^2 V_1 \rangle_{conn}^{TreeClosedString} \quad (1.1)$$

for a certain zero-momentum, possibly non-local, scalar pure-glue operator V_1 .

The lhs in the string interpretation is a sphere with k punctures and one open-string loop (hole) (Fig. 1), while the dual tree closed-string diagram is a sphere with k punctures and one cylinder attached to the hole boundary (Sec. 2).

Yet, building on the aforementioned renormalization properties [1], it is immediately clear that there is a tension between the UV log divergence in large- N QCD of the k -glueball amplitude with the insertion of one meson loop on the lhs of Eq. 1.1 and the universally believed UV finiteness [9, 10] of the tree 't Hooft planar pure YM diagram on the rhs of Eq. 1.1 that, being both a tree and a closed-string diagram, should be UV finite.

The canonical formulation of string theory resolves this tension in the general framework [6] and in explicitly solvable examples that bear deep analogies with the issue considered here [11]: the UV divergence of the geometrically planar open-string diagram on the lhs is mapped by open/closed duality into an infrared (IR) divergence of the dual tree closed-string diagram [6, 11] on the rhs (Sec. 2).

Yet, the IR divergence may occur only if the 't Hooft planar closed-string theory has no mass gap. Therefore, it is clear that this cannot work for large- N QCD [7] if it is assumed, on the basis of the overwhelming numerical evidence [12–15], that large- N QCD has a mass gap at tree level in the glueball sector, i.e. in the 't Hooft planar closed-string sector.

Indeed, we get immediately the following no-go theorem [7] under the assumption that the closed-string solution provides a dual UV-finite tree 't Hooft planar YM diagram on the rhs of Eq. 1.1, as it is universally believed [9, 10] and actually occurs in all presently known consistent string models for which there is an explicit realization of open/closed duality [11]: open/closed duality in a would-be canonical string solution of the 't Hooft large- N expansion for the QCD S matrix is incompatible with RG, AF and the existence of a mass gap in the tree closed sector, i.e. in the glueball sector at the leading $\frac{1}{N}$ order.

However, once the mass gap in the tree glueball sector is assumed, the only logically possible alternative to resolve the tension in the canonical string framework is that, conjecturally in non-trivial string backgrounds [5] realizing supposedly a QCD-like asymptotically free gauge theory with mass gap in the tree closed-string sector, the rhs of Eq. 1.1 may be UV log divergent rather than IR log divergent, thus resolving the tension by contradicting the universally believed UV finiteness of the dual tree closed-string diagrams.

In this paper we resolve this issue in QCD and QCD-like theories, working out the implications of a low-energy theorem of the Novikov-Shifman-Vainshtein-Zakharov type derived in [1] both in perturbation theory and in the large- N 't Hooft expansion.

It turns out that, in a perturbatively massless QCD-like theory [1], open/closed duality is consistent with perturbation theory at order of g_{YM}^2 since in Eq. 1.1 the operator implied by the perturbative low-energy theorem, $V_1 = \frac{1}{2} \int \text{Tr } F^2 d^4x$, is well defined at tree level and its insertion in the correlator is both UV and IR log divergent

because of conformal symmetry at order of g_{YM}^2 (Sec. 3).

Instead, at order of $\frac{N_f}{N}$ in the 't Hooft large- N expansion of QCD with massless or massive quarks, the operator provided by Eq. 1.1 and the low-energy theorem, $V_1 = N\beta_0^{NP} \log(\frac{\Lambda}{\Lambda_{QCD}^P}) \int d^4x \text{Tr } F^2$ + non-local UV-finite terms, is not well defined in the planar theory, but it is an infinite counterterm which diverges with the cutoff Λ before its insertion in the correlator, while the insertion of $\int d^4x \text{Tr } F^2$ is, in fact, UV finite because of AF (Sec. 4).

This is incompatible with the cutoff independence, and finiteness almost everywhere, of the string partition function viewed as a volume form on the moduli in the open side of the supposed duality (Secs. 2 and 4).

Thus, we get a new stronger version of the no-go theorem, based only on UV arguments, without assuming the existence of the glueball mass gap: RG and AF are incompatible with open/closed duality in a would-be canonical string solution of the 't Hooft large- N expansion for the QCD S matrix.

Moreover, the incompatibility extends to the large- N 't Hooft expansion of a vast class [1] of confining asymptotically free QCD-like theories including $\mathcal{N} = 1$ SUSY QCD.

2 Open/closed and UV/IR duality in a supposed string solution

The 't Hooft large- N expansion in YM theory and QCD [2] matches the topological expansion of a string theory with coupling $g_{Closed} \sim g_{Open}^2 \sim 1/N$, of closed strings in the glueball sector, and of open strings in the meson sector.

This matching has been suggesting the existence of a canonical string solution [3–5] of large- N QCD, and of confining asymptotically free QCD-like theories, for the last forty years.

Open/closed duality occurs canonically in string theory, for example, because the annulus diagram, i.e. a one-loop diagram in the open sector, is topologically the same as the cylinder, i.e. a tree diagram in the closed sector.

Moreover, there is a conformal map under which they are mapped into each other.

Thus, if conformal symmetry on the world-sheet is not anomalous, i.e. the string theory really exists, the annulus and the cylinder are identical, in the sense that the annulus in the open sector can be interpreted as the cylinder in the closed sector. As a consequence, in the examples worked out in [11], it holds, $\text{Tr}_{D_1 D_2} \exp(-t H_{Open}) \frac{dt}{t} = \langle D_1 | \exp(-\tau H_{Closed}) | D_2 \rangle d\tau$, as an identity between differential forms that represent the string partition functions viewed as volume forms on the moduli, for the annulus and cylinder respectively, with the duality map $\tau = \frac{1}{t}$ that exchanges the UV with

the IR [11], and $|D_1\rangle, |D_2\rangle$ certain D-brane states in the closed-string sector on the boundaries of the cylinder [11].

In fact, the duality between the annulus and the cylinder, and between the UV and the IR, is a particular case of a much more general correspondence [6]: "The world-sheet of a planar multi-loop diagram in open-string theory is conformally equivalent to a closed-string tree diagram. Indeed, all holes in the open-string diagram can be represented in the dual channel by means of external closed-string states, equal to the appropriate D-brane boundary state $|D\rangle$. Via this dual representation all potential UV divergences of the open-string diagram become equivalent to potential IR divergences due to on-shell closed-string states in the dual channel".

We consider now a punctured sphere with a hole (Fig. 1) that would correspond in the 't Hooft expansion of large- N QCD to a meson loop that renormalizes the glueball tree amplitudes [1].

Since the two string diagrams in Fig. 1 are conformally equivalent [6], and thus identical, if conformal symmetry on the world-sheet is not anomalous as it must be if a consistent canonical string solution of large- N QCD exists, open/closed duality requires the equality of the open- and closed-string partition functions viewed as volume forms on the moduli:

$$\begin{aligned} & \langle D_k(m_i) | Annulus(t) \rangle dm_1 \wedge \cdots \wedge dm_i \wedge \frac{dt}{t} \\ &= \langle D_k(m_i) | \exp(-\tau H_{Closed}) V_1 | 0 \rangle dm_1 \wedge \cdots \wedge dm_i \wedge d\tau \end{aligned} \quad (2.1)$$

for some smooth map $\tau = f(t) \sim \frac{1}{t}$ [6] (Sec. 2 and Fig. 1). $m_1 \cdots m_i$ are the remaining moduli of the Riemann surface obtained gluing D_k , a disk with k punctures in the interior, to an annulus with modulus t (i.e. a disk with one hole) in the open-string side of the duality, or to a cylinder with modulus τ in the closed-string side.

H_{Closed} is the world-sheet Hamiltonian in the closed sector. The subsequent integration on τ constructs the propagator of the closed string, $\int_0^\infty \exp(-\tau H_{Closed}) d\tau = H_{Closed}^{-1}$. The state $V_1 | 0 \rangle$ does not [11], and may not, depend on τ , because all the τ dependence must occur via the geometric generator of translations along the cylinder, H_{Closed} , in order for the tree closed-string propagator, H_{Closed}^{-1} , to arise from the τ integration.

Eq. 2.1 is a much stronger constraint than Eq. 1.1 which is implied by Eq. 2.1 after integrating on the moduli.

The densities, $\langle D_k(m_i) | Annulus(t) \rangle$ and $\langle D_k(m_i) | \exp(-\tau H_{Closed}) V_1 | 0 \rangle$, arise from the conformal field theory on the world-sheet underlying the string theory, and their existence means exactly that a canonical string solution for the large- N S matrix exists.

Because of their origin in a conformal field theory the densities are functions only of the moduli and of the physical parameters of the 't Hooft planar theory: in planar QCD, $\sqrt{T} \equiv \Lambda_{QCD}^P$ [1], and M_{PG}^P , the masses of the pseudo-goldstone bosons which

are functions of the renormalized quark masses, m^P . Therefore, both the densities depend on T and M_{PG}^P (or m^P), but may not depend explicitly on Λ .

A cutoff may be needed in the string S matrix only by integrating on the world-sheet moduli that in string theory play the analog role of Schwinger parameters of Feynman diagrams in field theory [9], since the densities, though well defined, need not to be integrable functions of the moduli.

Accordingly, $\langle D_k(m_i) | \text{Annulus}(t) \rangle$ is a function of the moduli that is Λ -independent and finite almost everywhere. Therefore, only after integrating on the moduli, specifically on the modulus of the open loop t , the lhs of Eq. 2.1 may be UV log divergent, since the string diagram represents a meson-loop correction to a k -point amplitude [1].

Correspondingly, if the open/closed duality holds, only after integrating on τ the dual closed-string diagram may be either IR or – contrary to the universal belief – UV log divergent.

3 Consistency of open/closed duality with perturbative renormalization in QCD-like theories

Initially, we suppose that the string solution reproduces perturbatively a QCD-like theory, i.e. the string coupling is identified with the gauge coupling, $g_{Open} \sim g_{YM}$ [11].

It is rather illuminating to check that the low-energy theorem [1] at order of g_{YM}^2 in QCD with massless quarks, and in a perturbatively massless QCD-like theory, is perfectly compatible with open/closed duality, as it occurs in the many examples in [11].

Firstly, we write the low-energy theorem [1] with the canonical normalization of the action:

$$\frac{\partial \langle F^2(z) F^2(0) \rangle}{\partial \log g_{YM}} = \int \langle F^2(z) F^2(0) \text{Tr} F^2(x) \rangle - \langle F^2(z) F^2(0) \rangle \langle \text{Tr} F^2(x) \rangle d^4x \quad (3.1)$$

Secondly, we employ a result in [16, 17] for the OPE of the operator $F^2(x) \equiv 2 \text{Tr} F^2(x)$ – that is well defined at tree level – at order of g_{YM}^2 :

$$\begin{aligned} F^2(z) F^2(0) &\sim \frac{N^2 - 1}{z^8} \frac{48}{\pi^4} (1 - 4\beta_0 g_{YM}^2 (\log \frac{1}{|z|\mu} - \log(\frac{\Lambda}{\mu})) + \dots) \\ &+ \frac{1}{z^4} \frac{4\beta_0}{\pi^2} g_{YM}^2 F^2(0) \end{aligned} \quad (3.2)$$

where we omitted the finite parts in the dots. Substituting Eq. 3.2 in the lhs of Eq. 3.1, we get:

$$2[\langle F^2(z) F^2(0) \rangle]^{1-Loop} = \frac{1}{2} \int [\langle F^2(z) F^2(0) F^2(x) \rangle]^{Order\ of\ g_{YM}^2} d^4x \quad (3.3)$$

Employing Eq. 3.2 and the OPE in the rhs of Eq. 3.1, we get:

$$\begin{aligned}
& 2\left[\frac{N^2-1}{z^8}\frac{48}{\pi^4}4\beta_0g_{YM}^2(\log(|z|\mu) + \log(\frac{\Lambda}{\mu}) + \dots)\right]_{div} \\
&= 2\frac{1}{2}\int\langle F^2(z)\frac{1}{x^4}\frac{4\beta_0}{\pi^2}g_{YM}^2F^2(0)\rangle^{Tree}d^4x \\
&= \langle F^2(z)F^2(0)\rangle^{Tree}\int\frac{4\beta_0}{\pi^2}g_{YM}^2\frac{1}{x^4}d^4x = \frac{N^2-1}{z^8}\frac{48}{\pi^4}8\beta_0g_{YM}^2\log(\frac{\Lambda}{\mu}) \quad (3.4)
\end{aligned}$$

where the factor of 2 on the rhs occurs because x may be close to 0 or to z .

We interpret Eqs. 3.3 and 3.4 in terms of open/closed duality.

The lhs is log divergent at one loop in perturbation theory because of the anomalous dimension, $\gamma_0 = 2\beta_0$, of $\text{Tr } F^2$. The divergence is both UV and IR because of the conformal symmetry of perturbation theory at order of g_{YM}^2 .

This is matched by the UV log divergence of a one-loop open-string diagram in the string examples [11] due to the same nontrivial one-loop beta-function coefficient, β_0 .

Moreover, the low-energy theorem constructs explicitly the closed-string operator, $V_1 = \frac{1}{2}\int \text{Tr } F^2 d^4x$, that enters the closed-string side of the duality in Eq. 1.1. On the rhs of Eq. 3.3 $\text{Tr } F^2(x)$ is a well-defined Λ -independent operator at the relevant order whose insertion at zero momentum is both UV and IR divergent, as predicted by open/closed duality, because of conformal symmetry of the OPE at the lowest non-trivial order.

In agreement with the IR divergence on the rhs, in all the consistent string models in [11] there is an IR log divergence in the closed sector (i.e. in the gravity sector) due to a massless dilaton [11], which is the string field dual to $\text{Tr } F^2$, that reproduces the very same beta function on the gravity side.

We should notice that everything that we mentioned is perfectly compatible with the first version of the no-go theorem recalled in Sec. 1 since there is no mass gap both in perturbation theory and in the aforementioned string models.

Most importantly, the log divergence arises on the rhs of Eq. 3.3 only after inserting the well-defined (at tree level) Λ -independent operator $\int \text{Tr } F^2 d^4x$ in the v.e.v.. This is consistent with the Λ independence of the closed-string partition function, viewed as a density on moduli spaces, as the log divergence may arise in the string diagram only after integrating on τ that constructs the tree closed-string propagator (Sec. 2).

Finally, the perturbative computation suggests that non-asymptotically-free theories (see Sec. 6 in [1]), which are asymptotically conformal in the UV or in the IR, may be compatible with the duality.

4 Inconsistency of open/closed duality with renormalization in large- N QCD-like theories

The situation is drastically different if it is assumed that the string solution reproduces 't Hooft topological large- N expansion, i.e. $g_{Closed} \sim \frac{1}{N}$.

In this case, in a perturbatively massless QCD-like theory, the low-energy theorem, within the leading $\frac{1}{N}$ accuracy, reads [1]:

$$\begin{aligned} [\langle \text{Tr } F^2 \cdots \text{Tr } F^2 \rangle^{\mathcal{N}P}]_{div} &= \frac{N\beta^P(g)\Lambda_{QCD}^{\mathcal{N}P}}{g^3\Lambda_{QCD}^P} \int \langle \text{Tr } F^2 \cdots \text{Tr } F^2 \rangle^P \langle \text{Tr } F^2(x) \rangle^P \\ &- \langle \text{Tr } F^2 \cdots \text{Tr } F^2 \text{Tr } F^2(x) \rangle^P d^4x \\ &\equiv \langle \text{Tr } F^2 \cdots \text{Tr } F^2 [V_1^P]_{div} \rangle^P - \langle \text{Tr } F^2 \cdots \text{Tr } F^2 \rangle^P \langle [V_1^P]_{div} \rangle^P \end{aligned} \quad (4.1)$$

Eq. 4.1 has precisely the structure to match Eq. 1.1, and it computes its divergent parts on both sides [1] due to the renormalization of Λ_{QCD}^P , with $[V_1^P]_{div} = -\frac{N\Lambda_{QCD}^{\mathcal{N}P}}{\Lambda_{QCD}^P} \frac{\beta^P(g)}{g^3} \int \text{Tr } F^2 d^4x = N[\beta_0^{NP} \log(\frac{\Lambda}{\Lambda_{QCD}^P}) + \cdots] \int d^4x \text{Tr } F^2$.

Moreover, in QCD V_1 can be derived directly from Eq. 1.1 by expanding at order of $\frac{N_f}{N}$ the QCD action, $V_1 = N_f \text{Tr} \log \frac{i\not{D}(A) - Z_m^P m^P}{i\not{D}(0) - Z_m^P m^P} = N\beta_0^{NP} \log(\frac{\Lambda}{\Lambda_{QCD}^P}) \int d^4x \text{Tr } F^2 +$ non-local UV-finite terms, with $Z_m \sim [\log(\frac{\Lambda}{\Lambda_{QCD}})]^{-\frac{\gamma_{0m}}{2\beta_0}} \sim Z_m^P(1 + \cdots)$, and $\gamma_{0m} = \frac{3}{(4\pi)^2}(1 - \frac{1}{N^2})$, where the dots stand for subleading log-log divergences arising at the order of $\frac{N_f}{N}$ due to the renormalization of Λ_{QCD}^P [1] and, by an analog direct computation, of Z_m .

Remarkably, within the leading-log accuracy, $[V_1^P]_{div} = [V_1]_{div}$ is the local counterterm, due to one meson loop [1] or to quark loops from the quark functional determinant, that produces the non-'t Hooft planar correction to β_0^P [1]. Besides, we will see below that the insertion of $\int \text{Tr } F^2 d^4x$ on the rhs of Eq. 1.1 is UV finite.

Thus, in QCD both with massless and massive quarks, the UV-finite non-local terms in V_1 may contribute on the rhs of Eq. 1.1 at most subleading UV log-log divergences. Instead, we see immediately that the obstruction to open/closed duality is the explicit quark-mass independent UV log divergence of the local part in V_1 before its insertion on the rhs of Eq. 1.1.

Indeed, by transferring into the would-be string solution the QCD result for V_1 on the rhs of Eq. 2.1, $\langle D_k(m_i) | \exp(-\tau H_{Closed}) V_1 | 0 \rangle$ is Λ dependent and UV log divergent – since $V_1 | 0 \rangle$ is created from the closed-string vacuum, $|0\rangle$, by an operator, V_1 , that is a log-divergent counterterm – that contradicts the almost everywhere finiteness and Λ -independence of the density on the lhs (Sec. 2). A similar argument holds for $\mathcal{N} = 1$ SUSY QCD.

Naively, there is a simple explanation for this obstruction to open/closed duality: since the closed-string tree diagrams are UV finite the only way for the rhs of Eq.

1.1 to be UV log divergent is that the boundary state, $V_1|0\rangle$, is explicitly UV log divergent, i.e. it is not well defined in the closed-string theory.

Besides, according to the naive expectation, the $\frac{\beta^P(g)}{g^3} \int \text{Tr } F^2 d^4x$ insertion is UV finite in YM theory, as follows from the asymptotically free RG-improved OPE worked out in [18, 19] within the leading and next-to-leading log accuracy, and afterwards in [20] within the leading-log accuracy:

$$\begin{aligned} \beta_0 F^2(z) \beta_0 F^2(0) &\sim \left(1 - \frac{1}{N^2}\right) \frac{1}{z^8} \frac{48\beta_0^2}{\pi^4} \left(\frac{1}{\beta_0 \log\left(\frac{1}{z^2 \Lambda_{QCD}^2}\right)}\right)^2 \\ &+ \frac{1}{z^4} \frac{4\beta_0^2}{\pi^2} \left(\frac{1}{\beta_0 \log\left(\frac{1}{z^2 \Lambda_{QCD}^2}\right)}\right)^2 \frac{\beta_0}{N} F^2(0) \end{aligned} \quad (4.2)$$

as opposed to lowest-order perturbation theory that is asymptotically conformal in the UV.

Indeed, the UV-divergent integral in perturbation theory on the rhs of Eq. 3.4, $\int \frac{1}{x^4} d^4x$, becomes the UV convergent integral in the 't Hooft planar theory, $\int \frac{1}{x^4 \log^2\left(\frac{1}{x^2 \Lambda_{QCD}^2}\right)} d^4x$, that follows from Eq. 4.2.

Summarizing, in the string interpretation on the lhs of Eq. 2.1 the density is finite almost everywhere and Λ independent, since it is a function only of T and M_{PG}^P , but the t integration, which constructs the meson loop, is UV log divergent.

On the contrary, on the rhs of Eq. 2.1 the density is UV log divergent before integrating on τ . Besides, after integrating on τ , the $\frac{\beta^P(g)}{g^3} \int \text{Tr } F^2 d^4x$ insertion is UV finite because of the AF.

Thus, Eq. 2.1, i.e. the open/closed duality, cannot hold.

Moreover, the asymptotic estimates in [1, 18–20] analog to Eq. 4.2 show that, because of the AF, no candidate Λ -independent, well-defined in the planar theory, scalar operator exists in large- N YM and $\mathcal{N} = 1$ SUSY YM theory – which would be necessarily local with dimension 4 since the log divergence arises as the open-string loop shrinks to a point – whose zero-momentum insertion may be UV log divergent in order to satisfy both Eq. 1.1 and Eq. 2.1, according to the universally believed UV finiteness of closed-string theories.

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